

Producer Theory Practice Set 4

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1. Consider two firms producing an identical good.

For firm 1, the total cost of q_1 units is given by $C_1(q_1) = q_1^2 + 4$

For firm 2, the total cost of q_2 units is given by $C_2(q_2) = q_2^2 + 4$

- If both firms produce a positive amount, determine how many units each firm must produce such that the production of the total amount, q , is efficient.
- Find the total cost of q units, $C(2,q)$, when the two firms produce a positive amount and the allocation of output among them is efficient. What are the marginal cost and average cost of the group of 2 firms?
- Suppose each firm is a price taker who can sell any amount it wishes at price p . Assuming each firm wishes to maximize profit, find each firm's supply function. What is each firm's minimum average cost? What is each firm's efficient scale of output?
- Find the aggregate supply function if these two firms are the only firms in the market. Can the total cost of the group, $C(2,q)$, you found in part (b) be derived from the aggregate supply of the two firms?
- If the price is $p=8$, how many units does each firm produce? What is the total amount produced? Is the total amount produced efficiently?

2. Repeat exercise 1 assuming that the total cost of firm 1 is $C_1(q_1) = q_1^2 + 4$
and the total cost of firm 2 is three times as large, that is, $C_2(q_2) = 3q_2^2 + 12$

3. Answer the following.

- a) For Jane's firm, the total cost of q units is given by $C(q) = 2\sqrt{q} + q$

What type of returns to scale does Jane's firm have?

Plot Jane's average cost and marginal cost and identify output intervals, if any, of IRTS, CRTS, DRTS.

- b) Suppose Dwane's firm is identical to Jane's firm. How should the production of a total of 400 units be divided between them so that the total cost of the 400 units is minimized? What would be the efficiency loss if Jane produces 360 units and Dwane produces the remaining 40 units?

4. Consider an industry of N identical firms where the cost function of each firm i , $i=1,\dots,N$, is given by

$$C_i(q_i) = 2q_i^2 + 8$$

- Determine how many units each firm must produce such that a total of q units is allocated efficiently among them. What is the marginal cost of this industry of N firms if total output is produced efficiently?
- Determine the efficient number of firms, N^* , required to produce a total of q units, and find the minimum industry cost of producing q units.

5. Consider an industry of identical firms where the production function of each firm is given by

$$q_i = f(L_i, K_i) = \begin{cases} 4 L_i^{(1/4)} & K_i = 2 \\ 0 & \text{otherwise} \end{cases}$$

where q_i , L_i , and K_i are respectively the output produced, labour and capital employed by firm i .

The firms can employ labour and capital at constant prices, given respectively by w and r .

Note that this specification of the production function simply says that every firm producing a positive amount of output must employ exactly 2 units of capital (and of course the required amount of labour). Therefore if there are N operating firms, the total amount of capital employed is $K = 2N$.

- Derive the cost function of firm i .
- Derive the supply function of firm i , assuming the firm is a price-taker who maximizes profit.
- Suppose the price of capital increases. How will this affect the the supply of the firm. Plot the supply of the firm before and after the increase in r .
- Derive the minimum industry cost of producing a total output of q units with N firms.
- Derive the efficient number of firms, N^* , and then the minimum industry cost of producing q units.
- Derive the two-input production function, $q = F(L, K)$, representative of how the *industry* combines the two inputs to produce output.

6. Suppose the supply function of an increasing cost industry can be approximated quite accurately by the function

$$q = \begin{cases} -80 + 400p & \frac{1}{5} < p \\ 0 & \text{otherwise} \end{cases}$$

where p denotes the good's price and Q is the quantity supplied by the industry.

- Find and plot the industry's marginal cost.
- What is the total industry cost of q units of output?
- How many units will the industry supply if $p = 10$? What are the industry's total cost and total profit at that price?

7. Consider another increasing cost industry with supply function

$$q = \begin{cases} -80 + 800p & \frac{1}{10} < p \\ 0 & \text{otherwise} \end{cases}$$

- Find the expression which gives the price elasticity of the industry's supply as a function of price.
- What happens to the price elasticity of supply as price increases?
- Call this industry "industry 2" and the industry in the previous question "industry 1". Which industry's supply is more responsive to price (at any price where both industries supply a positive amount)?